Relativistic free-particle quantization on the light-front: New aspects

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Abstract. We use the light-front machinery to study the behavior of a relativistic free particle and obtain the quantum commutation relations from the classical Poisson brackets. We argue that the usual projection onto the light-front coordinates for these from the covariant commutation ralations does not reproduce the expected results.

LIGHT-FRONT QUANTIZATION

The Lagrangian for a free relativistic particle of mass m, is given by [2] $\mathscr{L} = -m\sqrt{\dot{x}^2}$. The canonical four momentum p^μ is obtained directly from

$$p^\mu = \frac{\partial \mathscr{L}}{\partial \dot{x}_\mu} = \frac{-m\,\dot{x}^\mu}{\sqrt{\dot{x}^2}} \tag{1}$$

and the fundamental Poisson brackets are:

$$\{x^\mu, p_\nu\} = \delta^\mu_\nu \tag{2}$$

and

$$\{x^\mu, x_\nu\} = 0 = \{p^\mu, p_\nu\} \tag{3}$$

Canonical quantization can not be obtained directly from these brackets through the rules of quantization, because this theory has constraints [3]. From (1), we get immediately that

$$\phi_1 = p^2 - m^2 \approx 0 \tag{4}$$

Using this basic result in

$$\left\{x^\mu, p^2 - m^2\right\} = \left\{x^\mu, p^2\right\} - \left\{x^\mu, m^2\right\} \tag{5}$$

we have (observing that the second term on the right hand side yields zero straightforwardly)

$$\left\{x^\mu, p^2\right\} = 2p^\nu \left\{x^\mu, p_\nu\right\},$$

which with (3) gives,

$$\left\{x^{\mu}, p^{2}\right\} = 2p^{\mu}. \tag{6}$$

For the component $\mu = 0$ we have

$$\left\{x^{0}, p^{2}\right\} = 2p^{0} = 2E, \tag{7}$$

that is, for this particular time component the Poisson bracket relates to the total energy of the system. In the case of $\mu = +$ coordinate we have

$$\left\{x^{\mu}, p^{2}\right\}^{lf} = p^{+}g^{+-} = 2p^{+} \tag{8}$$

This result agrees perfectly with the covariant case (6). From Eq.(4) transformed into the light front

$$\phi_1 = p^{+}p^{-} - p^{\perp 2} - m^2 \approx 0 \tag{9}$$

we have the primary constraint. The light front Lagrangian is

$$\mathscr{L}_{lf} = -m\sqrt{\dot{x}^{\mu}\dot{x}_{\mu}} = -m\sqrt{\dot{x}^{+}\dot{x}^{-} - \dot{x}^{\perp 2}}. \tag{10}$$

The canonically conjugate momentum components are

$$p^{-} = -m\frac{\dot{x}^{-}}{\sqrt{\dot{x}^{2}}}, p^{+} = -m\frac{\dot{x}^{+}}{\sqrt{\dot{x}^{2}}} \tag{11}$$

and

$$p^{\perp} = -m\frac{\dot{x}^{\perp}}{\sqrt{\dot{x}^{2}}}. \tag{12}$$

The Hamiltonian is

$$H_c^{lf} = p\,\dot{x} - \mathscr{L}_{lf} = 0 \tag{13}$$

This result gives us an indication that we use

$$\widetilde{H}^{lf} = \lambda\left(p^{+}p^{-} - p^{\perp 2} - m^2\right)$$

which by the condition that the constraint does not evolve in time we have

$$\dot{\phi}_1 = \left\{\phi_1, \widetilde{H}^{lf}\right\} \approx 0. \tag{14}$$

This means that there are no new constraints and we are unable to determine the multiplier λ.

Let us then, as before impose a new constraint in the light front

$$\phi_2 = x^{+} - s \approx 0, \tag{15}$$

so that,

$$\{\phi_1,\phi_2\} = -2p^+ \tag{16}$$

Comparing with the result (7) we again perceive that there is an inconsistency here. Instead of the energy p^- we get the momentum p^+.

Constructing the Hamiltonian and with the condition of non evolution in time of the constraint we have

$$H = \frac{1}{2p^+}\left(p^+p^- - p^{\perp 2} - m^2\right) \tag{17}$$

In order to obtain the Dirac brackets, let us construc the matrix C, where $C_{ij} = \left\{\phi_i,\phi_j\right\}$ and $i,j = 1,2$. The Dirac brackets then is given by

$$\{x^\mu,p^\nu\}_D^{lf} = g^{\mu\nu} - \{x^\mu,\phi_1\}C_{12}^{-1}\{\phi_2,p^\nu\} - \{x^\mu,\phi_2\}C_{21}^{-1}\{\phi_1,p^\nu\}\,.$$

Quantization is now achieved by

$$[x^\mu,p^\nu]_D^{lf} = i\left[g^{\mu\nu} - \left(p^+g^{\mu-} + g^{\mu+}p^- - 2p^\perp g^{\mu\perp}\right)\frac{1}{2p^+}g^{+\nu}\right] \tag{18}$$

Clearly we have here the problem of zero modes in the light-front.

CONCLUSION

We have (6) for the component $\mu = 0$ we have $\left\{x^0,p^2\right\} = 2p^0 = 2E$, that is, for this particular time component the Poisson bracket relates to the total energy of the system. For the light-front case, $\mu = +$ and therefore $\left\{x^+,p^2\right\} = 2p^+$. Which means that the light-front "time" variable x^+ relates to the momentum p^+, an apparent inconsistency between canonically conjugate variables. In the covariante case we have a correlation between time and energy in the Poisson brackets, while in the light front this correlation is lost. See Eq.(8). This conclusion can be the origin to problem of zero modes in the light-front. In the paper [4], we propose a treatment for the presented inconsistency

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REFERENCES

1. P.A.M.Dirac, Rev.Mod.Phys.**21**, 392 (1949).
2. L.D.Landau and E.M.Lifshitz, Classical Theory of Fields, Ed. Mir Moscou (1972).
3. P.A.M. Dirac, Canad. J. Math. **2** (1950) 129. P.A.M. Dirac, Canad. J. Math. **3** (1951) 1.
4. A. T. Suzuki, J. H. O. Sales and G. E. R. Zambrano, hep-th/0405095.